



# Hopping, Bouncing and Jumping for Variables

Students participate in activities that provide a conceptual understanding of the role of variables in expressions and equations.

**Grade Levels:** 5<sup>th</sup> - 7<sup>th</sup>

**Topics:** Variables, Variable expressions, Equations, Data Analysis

## Goals:

- Students will have a conceptual understanding of the role of a variable in expressions and equations.
- Students will be able to interpret variable expressions and evaluate them for a given value of the variable.
- Students will be able to write variable expressions to describe a real-life situation.
- Students will write and solve one-variable equations.
- Students will describe data with measures of central tendency.

**Pre-Requisite Knowledge:** order of operations

**Common Core Math Standards:**

5.MD.2	6.EE.2	6.SP.2	7.EE.4	7.SP.1
	6.EE.3	6.SP.3		7.SP.2
	6.EE.5	6.SP.4		
	6.EE.6	6.SP.5		
	6.EE.7			

## Materials:

- 3 paper bags
- Balls to bounce (2 or 3; tennis balls recommended)
- Jump ropes (2 or 3)
- Slips of paper and tape– one per student (may also use large sticky notes)
- Stop watches or timers
- Small white boards and dry erase markers
- Linking cubes – 10 per student
- Large chart paper
- Manila paper
- Markers
- Student Activity Sheets
  - “Variable Expression Charts”
  - “The Great Jump Rope Mysteries”
  - “Team Time”
  - “How Fast Can You Stack?”
- 3" x 5" cards labeled with variable expressions – one per student
- 5" x 8" cards for students
- Adding machine tape (optional)

**Preparation Time:** 30 minutes – 1 hour

**Activity Time:** 5 – 7 lessons

**Reference:**

1. Knuth, Alibali, Hattikudur, McNeil, Stephens; “The Importance of the Equal Sign Understanding in the Middle Grades,” *Mathematics Teaching in the Middle School*, Vol. 13, No. 9, May 2008, pgs. 514-519

**Additional Resources:**

1. NCTM Illuminations interactive computer activities – Pan Balance Numbers and Pan Balance Shapes

## Lesson Plans

**Lesson One: Students understand that variables represent unknown numbers. Students begin to evaluate expressions for given variable values.**

Label the three paper bags as HOP, JUMP, and BOUNCE. Hand each student a slip of paper and have them write their name on it. Explain that they will be able to choose one of the three activities: (1) hopping on one foot, (2) jumping rope, or (3) bouncing and catching a ball. **Note: If the teacher prefers, students can hop on one foot, or do jumping jacks.**

Students will put names in the bag of choice. The teacher will draw three names from one bag. These three students will go to the hallway and count the number of times they are able to hop, jump or bounce for a given time, such as 30 seconds or 1 minute. Students need to count for themselves after the timer begins and stop when the timer rings. (As an option, a fourth student can go to the hall to be the timer.)

While students are in the hall, ask the class for predictions and write them on the board.

When students return, each one will whisper to the teacher the number of times they performed the task. Hand each student a white board and write a variable expression on each person's board. Make sure one person is just a single letter variable and that all expressions use the same variable. See the example below:

Mary hopped 21 times	Board says $m$
Latonia hopped 25 times	Board says $m + 4$
Brett hopped 17 times	Board says $m - 4$

Have the three students stand in front of the class and hold their boards so the class can see the expressions. Ask the class if they know what the variable represents (number of times Mary hopped). Emphasize that the variable represents **a number**. Ask students to put the students in order from least to greatest number and have them explain how they know this is correct. Try to elicit comments such as, “Latonia did 4 more hops than Mary,” etc.

Ask students if they could figure out the number of times Latonia and Brett hopped if they knew Mary's number. Tell them Mary's number and have them pair share their ideas for Brett and Latonia. Discuss ideas as a class.

Hand out student activity sheet “Variable Expression Charts.” Have a large sheet of chart paper labeled in the same way. Write the expressions on chart paper. Have students pair share ideas for what each expression means in words. Discuss this as a class and write ideas on the chart. In the example

above,  $m + 4$  could mean: “4 more than a number,” “a number increased by 4,” “4 added to a number,” “a number plus 4,” “4 more than the number of times Mary jumped,” etc. Have students evaluate the expressions for the given value of the variable.

Select three more names from another bag and continue until all students have taken a turn.

**Note:** Be sure to include variable expressions that have multiplication and division and parentheses. In the example above, you could have:  $b$  represents Brett's number,  $2b-9$  represents Latonia's number and  $3(b-10)$  is Mary's number.

## Lesson Two: Students write and evaluate variable expressions.

As a review of lesson one, write the following three numbers 12, 20 and 18 across the top of the board. Have a 5" x 8" card with the letter  $x$ . Tape the letter under the number 12. Have students work with partners to write variable expressions that describe 20 and 18 compared to  $x$ . Have students read their expressions and write them below the numbers. Ask students how they would know if the expressions are correct. If needed refer to the chart created in class in lesson one and have students review the idea of *evaluating a variable expression*.

Repeat this process, but move the  $x$  under a different number.

Change the three numbers to 7, 15, and 40. Place the  $x$  under the 7. Encourage students to think of using multiplication as well as addition as they write the variable expressions. Move the variable as done above.

Hand out 3" x 5" cards with variable expressions. Each expressions should use the same variable. Some suggested expressions are:  $3n$ ,  $2n$ ,  $2n+1$ ,  $n - 6$ ,  $3n - 4$ ,  $3n+2$ ,  $n / 4$ ,  $n+11$ ,  $5n$ ,  $2(n+2)$ ,  $n$ ,  $n - 7$ ,  $\frac{1}{2}n$ ,  $4n - 3$ .

Tell students that you will be giving them a value for the variable  $n$  (for example, “ $n$  is worth 8”). The class will then have one minute to put themselves in order from least to greatest without talking. A fifth grade class would use all positive whole numbers. Sixth and seventh graders could begin with a positive whole numbers, but then try a fraction and a negative integer. To assess, each student would state their expression in words and explain the value of the expression for the given variable.

Have students work with a partner on student activity “The Great Jump Rope Mysteries.”

Discuss student work as a class.

Have students write their own problem on a 5" x 8" card with solutions on the back. Collect as a formative assessment.

## Lesson Three: Students write and evaluate expressions and equations.

Divide the class in to teams of four students. Hand out student activity sheet “Team Time” which requires students to collect data from their team and write variable expressions.

On a large sheet of chart paper, have a the following table ready for students to complete:

Student	# of jumping jacks	# of hops	# of bounces	# of jumped ropes

Discuss each team's ideas after students have completed part one.

Select three students' data from the table in one column. Write the numbers on the board as

done in lesson two. Once again select one number to be the  $x$ . Have students help to write variable expressions for the other two numbers. Hand out white boards and dry erase markers and have students work with their partner to write an equation that shows the total number of hops students.

Example:	Board:	Student Work:
	20 25 18	$20 + 25 + 18 = 63$
	$x+2$ $x+7$ $x$	$x+2 + x+7 + x = 63$
		$3x + 9 = 63$
		check: $3(18) + 9 = 63$

Challenge students to find other equal relationships between the numbers. For example,  $18 \times 2$  is the same as  $25 + 11$ . They could then write  $x + 7 + 11 = 2x$ .

Have students complete part two of the activity sheet independently and then compare and discuss their ideas with their team.

#### **Lesson Four: Students create data in order to write expressions and equations.**

Students will each be given 10 linking cubes and student activity sheet “How Fast Can You Stack?” Each student will race to build a single tower of cubes as quickly as possible. Each student is responsible for recording their own time. Call out each second as it passes. Conduct several trials. Students should follow the directions on the activity sheet. Answers will vary. It may be helpful to have an overhead of the activity sheet for a discussion with the class.

Have students use manila paper to create posters of their variable expressions and equations. Hang these around the room. Hand out sticky notes and have students complete a carousel activity: assign students into small groups and allow them to study each poster; students should write helpful comments on the sticky notes and stick them to the posters. Challenge the students to find the value of each variable in order to determine the number of seconds classmates took to stack ten cubes.

**Note:** If possible, following this lesson, have students try the computer activity “Pan Balance Shapes” on NCTM’s Illuminations website. <http://illuminations.nctm.org/ActivityDetail.aspx?ID=33>

#### **Extension: Data Analysis**

Create a line plot of the class data from lesson four and use it to find mean, median and mode. It can be interesting to create a large line plot using adding machine tape and sticky notes for each student. Discuss which measure of central tendency best describes the data. Answers will vary, but if there are significant outliers the mean is distorted and the median is a better indicator of what happened in the class. Challenge them to arrange the sticky notes differently to achieve the same mean with different data. Discuss whether the class results would be a good sample of expected outcomes for other students in the same grade. It would be interesting to have different classes try this activity and then compare results. Discuss whether the class data would be a good sample of expected outcomes for students in the whole school. If the school has fifth through eighth grade, students would probably determine that the sample of one grade may not be accurate for the school.

## Lesson Five: Students write and solve equations based on a story.

As a review of the previous lessons, hand out the 3" x 5" variable expression cards used in lesson two. Tell students that they need to determine the value of the variable when you tell them the value of the expressions. Each expression will have the same value. Have students write their equation and solution on a white board. For example, each expression could be worth 15. Then  $3n=15$  means  $n=5$ ;  $2n+1=15$  means that  $n=7$ ;  $\frac{1}{2}n=15$  means that  $n=30$ . As an option, have cards taped to the board that are possible values of the variables. As students find the value, they can go to the board to take the number they think is correct.

Discuss student strategies for finding the value of the variables. Accept all reasonable explanations.

Tell students that they are going to use the skills they've learned to solve a mystery.

Have the following "mystery" on chart paper. Allow students time to work independently and then have them pair share.

*Four students, Amy, Becky, Clyde and Dwayne stacked ten cubes as the teacher timed them. Amy took twice as many seconds as Becky. Clyde had the same time as Becky. Dwayne took three seconds longer than Amy. Their total time was forty five seconds. How many seconds did each student take?*

Allow students to use any problem solving strategy. Encourage them to think about variables and equations. One possible solution is below:

***b represents the number of seconds Becky needed***

***b also represents the number of seconds Clyde need***

***2b is the number of seconds Amy needed***

***2b+3 is the number of seconds for Dwayne***

***Together the equation is  $45=b+b+2b+2b+3$***

***So,  $45=6b+3$***

***Thus,  $b=7$***

***Therefore, Becky and Clyde each took 7 seconds; Amy took 14 seconds; and Dwayne took 17 seconds.***

***Check:  $7+7+14+17=45$***

Post this second "mystery" for students to solve.

*Evelyn and Frank each required the same number of seconds to stack their ten cubes. Evelyn noticed that three times the number of seconds she had minus five was the same as twice the number of seconds Frank had plus four. How many seconds did it take Evelyn or Frank to stack ten cubes?*

Again, give students time to use any problem solving strategy, but encourage algebraic statements. One possible solution is:  $3x - 5 = 2x + 4$ , where  $x$  represents the number of seconds that Evelyn or Frank used to stack ten cubes. Using guess and check or transformations of equations,  $x=9$ .

Students should now work with a partner to create their own mystery. Students do not have to be limited to the cube stacking activity. Provide manila paper and markers so students can enhance their problems with illustrations. All solutions must be provided on a separate 3" x 5" card. Students can take turns in front of the class presenting mysteries and challenging classmates or problems can be exchanged and solved between teams.