Fermi Questions

What would it take to fill this room with popcorn? How many people in the world are talking on their cell phones at this instant? If everyone in the country needed to be innoculated against a virulent strain of influenza, how quickly could this be done? Find the answers to these and other seemingly inaccessible questions by estimating and using knowledge gained from everyday experiences.

Levels

- Grades 4 through 12
- Adults will find this activity interesting
- This activity can be extended to an undergraduate research project

Topics

- modeling
- estimation
- measurement
- algebraic expressions
- formulas
- problem solving

Goals

- Identify and evaluate modeling strategies
- Generate many potential solutions to a given problem
- Apply principles and generalizations to new problems and situations
- Analyze problems from different points of view
- Make decisions as part of a group
- Communicate mathematical ideas verbally and in writing
- Work with a variety of measurement tools
- Create and utilize formulas to tackle a problem
- Use internet search tools to find information
- Hone estimation skills and confidence

Prerequisite Knowledge

- familiarity with basic measurement skills
- familiarity with common uses of addition, subtraction, multiplication, and division
- familiarity with fractions, decimals, and percents

Preparation Time 5 to 15 minutes

Activity Time Usually 1 to 4 hours per Fermi question

Materials & Preparation

- journal or paper for recording findings
- pencils
- copies of handouts (optional)
- appropriate measurement tools (optional)
- appropriate equipment for gathering experimental evidence (optional)
- access to the internet (optional)

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Credits

Sample Fermi questions were gleaned from the following sources:

- Teachers and students from the greater South Bend, Indiana area
- *Fermi Questions* web page created by Louisiana Lessons in collaboration with the Math Forum; http://mathforum.org/workshops/sum96/interdisc/sheila1.html
- *Fermi Questions* web page created for a Science Olympics competition by the Department of Physics at the University of Western Ontario; http://www.physics.uwo.ca/science_olympics/events/puzzles/fermi_questions.html
- *On Beyond a Million: An Amazing Math Journey* by David M. Schwartz includes a broken popcorn machine in the story as a launch into large numbers. Elementary students might enjoy this book as a launch into popcorn Fermi questions.

Fermi Questions Lesson Plan

Introducing Fermi Questions

Distribute the Fermi Questions handouts to the students. Briefly introduce Enrico Fermi and Fermi Questions by reading and discussing the introductory page together. Explain that they will relate seemingly complicated questions to their everyday experiences. They will estimate by making a series of simple assumptions to arrive at a reasonable solution.

You can opt to choose an activity ahead of time, allow students to choose a topic as a whole group, allow small groups to create their own questions, or give students a limited set of options to choose from. These choices have different implications for the amount of time the activity will take and what materials might be needed.

Fermi Question Lab

The lab outlined here asks students to complete six steps for each Fermi Question:

1. Question: State the question and clarify the interpretation.
2. Wild Guess: Make a wild guess involving no calculations.
3. Educated Guess: Make an educated guess involving a chain of reasoning and calculations based on everyday experiences and estimates.
4. Variables and Formulas: Define variables and create a formula to solve the Fermi question.
5. Gathering Information: Perform experiments, conduct surveys, make measurements, and search for information to improve estimates and to find a smallest reasonable value, a largest, reasonable value, and a most likely value for the answer to the Fermi Question.
6. Conclusions: Summarize the overall conclusions, possible sources of error, interesting facts learned, possible directions for future investigation.

Providing guidance

Walk around and listen to students as they discuss and work through the problems, providing guidance as necessary. If students need more support, stop them after each step and have them share their work so far. Depending on the level of the students, it may be helpful to have each group turn in their work following each step so that you can verify that they are on the right track. This can also break up the process into smaller chunks of time.

Presentations

A project of this type is a great opportunity to have students practice their written and verbal communication skills. Students often enjoy making a poster showing their findings, making a power point presentation, or creating a group report using a blog or a collaborative editor.

Examples Illustrating How To Make Educated Guesses

Here are sample reasoning processes for several Fermi Questions. Note that some of the estimates may not be accurate. The people making these estimates will need to gather additional information.

How many bricks are in the exterior of our school building?

I think that each brick is about 6 inches long and about 3 inches high. I think that the school is about the length of a football field on each side. A football field is 100 yards or 300 feet. It would take 600 bricks to equal this length on each of the four sides of the building. I think the school is about 30 feet tall. It would take 4 bricks for each foot, so that means the school is about 120 bricks high. So each of the four sides of the school needs about $600 \times 120 = 72,000$. This means there are about 288,000 bricks.

What is the volume of air that I breathe in one day?

It takes about 10 breaths to blow up a balloon the size of a two-liter bottle. So, that means I breathe about one liter of air for every five breaths.

I breathe about 10 times every minute, so I breathe about two liters of air every minute. This means that in an hour, I breathe about 120 liters of air. So each day, I breathe about 2,880 liters of air.

How many kernels of popcorn would it take to fill this classroom?

I think that a puffed kernel of popcorn occupies less than a cube which is a half inch on each side. This means that 8 pieces of popcorn should occupy each cubic inch. There are $12 \times 12 \times 12$ cubic inches in a cubic foot. I will approximate that as $10 \times 10 \times 10$ cubic inches since I am just estimating anyway. That means that there are about 1000 cubic inches in a cubic foot and about 8 pieces of popcorn in each cubic inch, so I have about 8,000 pieces of popcorn in each cubic foot.

The square ceiling tiles in our classroom seem to measure about 2 feet on each side. The room is 25 tiles long and 25 tiles wide, so the length and width of the classroom is approximately 50 feet by 50 feet for 2500 square feet. I think the classroom is probably about 2 of me tall, so the ceiling might be about 10 feet high. This gives a volume of about 25,000 cubic feet.

So about $25,000 \times 8,000 = 200,000,000$ kernels of popcorn would be required to fill the room.

How many people in the world are talking on their cell phones in any given minute?

I think that about half the people in the world have cell phones and my guess is that there are about 6 billion people in the world now. That means that 3 billion people have cell phones.

I use my cell phone for a total of about one hour each day. Some people use their phones more than I do and some people use their phones less than I do. That means that I use my phone about 1/24 of the minutes in a day, which I will round to 1/25 of the minutes in a day. If I divide the 3 billion people with phones by 25, I should obtain a rough count of the number of people using a phone in any given minute. This means that about 120,000,000 people world wide are using a phone during any given minute.

How many pennies would need to be stacked to reach your height, the height of the school, the tallest building in the world, Mount Everest, outer space?

- I think that it takes about 4 penny rolls to equal one foot. I am about 5 feet tall, so that is about $5 \times 4 = 20$ penny rolls. Each roll has 50 pennies. So that is about $50 \times 20 = 1000$ pennies to equal my own height. (About $10.00 in pennies.)
- I think that the school is two stories tall and that each story is about 15 feet tall. That means that the school is 6 times as tall as I am and so it would take about 6000 pennies to be the size of the school (about $60.00 in pennies).
- One hundred stories is more than most tall buildings have, so $3000 \times 100 = 300,000$ (about $3,000 in pennies) is probably enough pennies to surpass the tallest building.
- Denver is the mile high city, and I know that the Rockies go up at least another mile. I don't know how the Rockies compare with the Himalayas where Mount Everest is, but let's say that they are twice the elevation. That would be four miles high. There are more than 5,280 feet in a mile, so I would need more than 21,000 feet of pennies to reach four miles. That means about 84,000 penny rolls or about 4,200,000 pennies ($42,000 in pennies).
- How far up is outer space? I am not really sure, but I think 100,000 feet (roughly 19 miles) is about right. So that would be about 20,000,000 pennies to reach to outer space (a mere $200,000 in pennies).

Example of a Full Fermi Lab Solution

1. Question:

The question I am choosing is "If I combine all of the liquid I will drink over my lifetime, how many baths would it fill?" I am interpreting this to mean liquid that I drink from a cup of some kind. I am not including liquid in soup, fruit, or other foods. I am assuming that the bath tub is filled completely.

2. Wild Guess:

If I just make a wild guess, I think that the answer might be about 5,000 bath tubs of liquid. Making the wild guess is not very satisfying because I have no idea whether it is reasonable or not.

3. Educated Guess:

Assumptions:

I can make a more educated guess by making some assumptions.

- Today, I had 4 mugs of coffee (about one and one half pints) two glasses of orange juice (half a pint), a can of soda (about half a pint), some milk on my cereal (about a third of a pint). I must have missed something . . . so I shall write down this assumption: On a typical day I drink about 3 pints of liquid.
- Now I also need to know about bathtubs. I am 6 feet tall, and when I soak in the bathtub, I can reach the taps with my toes while keeping my head above water. So the bath must be about 5 feet long. The tub is about 2 feet 6 inches wide on the inside, and about 1 foot deep. Using the formula for the volume of a rectangular solid, I can make my second assumption: A full bath holds about $V = L \times W \times H = 5 \times 2.5 \times 1 = 12.5$ cubic feet.
- One last assumption: I will live about 75 years.

Calculations:

The units I have chosen are incompatible. I've got pints and cubic feet. This is where I need a reference book. It says that 1 US pint = 29 cubic inches I know that 1 cubic foot $= 12 \times 12 \times 12 = 1728$ cubic inches. (12 inches are in a foot.) So, lets change all the units to cubic inches:

- I drink about $3 \times 29 \approx 90$ cubic inches per day. (Notice that I rounded my answer because I am approximating anyway.)
- My bath holds $12.5 \times 1728 \approx 22,000$ cubic inches.
- In 75 years that is $90 \times 365 \times 75 \approx 2,500,000$ cubic inches. So that means I will drink about $2,500,000 \div 22,000 = 113$ bath fulls of liquid.

Answer:

In a lifetime I will drink a little over 100 bath fulls. That answer strikes me as surprisingly low because that means that I only drink about $1\frac{1}{2}$ bath fulls of liquid each year. Perhaps some of my estimates were off a bit or perhaps my sense of how many bathtubs of liquid I drink is not accurate. On the other hand, I see now that my wild guess of 5,000 bathtubs of liquid is too high, since that would mean that I drink $5000 \div 75 \approx 67$ bathtubs of liquid each year or one bathtub full every 5 or 6 days.

4. Variables and Formulas:

Here are the variables I used while estimating the answer.

- Let C_{P} be the average number of pints I consume each day.
- Let C be the average number of cubic inches I consume each day.
- Let T be the total number of cubic inches I will consume over my lifetime.
- Let L be the length of the inside of the bathtub in inches (since I ended up converting).
- Let W be the width of the inside of the bathtub in inches.
- Let H be the height of the inside of the bathtub in inches.
- Let V be the volume of the bathtub in cubic inches.
- Let Y be the number of years that I will live.
- Let D be the number of days that I will live.
- Let B be the number of bathtubs of liquid I will consume over my lifetime.

Now I can use these variables to write the formulas used to calculate the answer.

- $C = 29C_{\mathrm{P}}$ (This formula converts the number of pints I consume each day to the number of cubic inches I consume each day.)
- $V = LWH$ (This formula finds the volume of the bathtub in cubic inches.)
- $D = 365Y$ (This formula gives the total number of days that I will live.)
- $T = CD$ (This formula gives the total number of cubic inches I will consume over my lifetime.)
- $B = \frac{T}{V}$ (This formula divides the total amount of liquid I will consume over my lifetime by the amount of liquid held by one bathtub to get the number of bathtubs of liquid I will consume over my lifetime.)

Notice that I could combine all these small formulas together into a single formula,

$$B = \frac{(29C_{\mathrm{P}})(365Y)}{LWH}$$

which simplifies to

$$B = \frac{10585C_{\mathrm{P}}Y}{LWH}.$$

This formula requires that I estimate or measure how many pints we drink each day on average, how many years I will live, and the three dimensions of a bathtub to find the total number of bathtubs of liquid I will consume over my lifetime.

5. Gathering More Information

There are several measurements and pieces of information that I could gather to improve the estimate.

- I could measure the length, width, and height (in inches) of my bathtub.
- I could look up the average lifespan (in years) of people in the United States.
- I could keep track of how much liquid I consume every day for a week and take an average.

The formula I found in the previous step would then make it easy to obtain a revised answer.

Data

The actual length in inches of my bathtub is $L = 52$.
The actual width in inches of my bathtub is $W = 21$.
The actual height in inches of my bathtub is $H = 13$.
The average lifespan (in years) of people in the US is $Y = 78.11$. (Source: CIA World Factbook)

I tracked my liquid consumption for a week and obtained the following data.

Sunday: 10 cups of liquid = 5 pints
Monday: 6 cups of liquid = 3 pints
Tuesday: 7 cups of liquid = 3.5 pints
Wednesday: 12 cups of liquid = 6 pints
Thursday: 4 cups of liquid = 2 pints
Friday: 8 cups of liquid = 4 pints
Saturday: 5 cups of liquid = 2.5 pints

Based on this sample, I drink an average of about 3.7 pints each day, so $C_{\mathrm{P}} = 3.7$.

Best Computed Answer

The formula that I found earlier tells me that the number of bathtubs of liquid consumed in my lifetime can be computed using the following formula:

$$B = \frac{10585 C_{\mathrm{P}} Y}{LWH}.$$

So my best estimate for the number of bathtubs of liquid I will consume during my lifetime is:

$$B = \frac{10585 \cdot 3.7 \cdot 78.11}{52 \cdot 21 \cdot 13} \approx 215.5$$

Largest and Smallest Possible Values

Now I will investigate what the largest and smallest values

Bathtubs come in different sizes, but I could decide what the smallest and largest dimensions would be. It should at least be possible to sit down in a bath tub, so the smallest dimensions might be 30 inches by 30 inches by 12 inches deep. Large bathtubs can be pretty big, but let's just say for the sake of argument that the bathtub is at most 6 foot by 6 foot by 3 feet, or 72 inches by 72 inches by 36 inches.

People live different numbers of years, but I know that I have already lived 35 years and I know that I am unlikely to live longer than 110 years.

The largest amount of liquid I can imagine drinking in one day is 2 gallons (or 16 pints). I think that I would need to drink at least 2 pints a day on average.

To find the smallest possible answer, I should use the smallest possible numbers for the number of pints consumed and the number of years lived, and the largest possible numbers for the length, width, and height of the bath tub. If I do this, I find that

$$\begin{aligned} B &= \frac{10585 C_{\mathrm{P}} Y}{LWH} \\ &= \frac{10585 \cdot 2 \cdot 35}{72 \cdot 72 \cdot 36} \\ &= \frac{740950}{186624} \\ &\approx 4 \end{aligned}$$

So the smallest reasonable estimate is 4 (very large) bathtubs of water consumed over the course of my lifetime.

To find the largest possible answer, I should use the largest possible numbers for the number of pints consumed and the number of years lived, and the smallest possible numbers for the length, width, and height of the bath tub. This gives

$$\begin{aligned} B &= \frac{10585 C_{\mathrm{P}} Y}{LWH} \\ &= \frac{10585 \cdot 16 \cdot 110}{30 \cdot 30 \cdot 12} \\ &= \frac{18629600}{10800} \\ &\approx 1725 \end{aligned}$$

So the largest possible answer should be 1725 (very small) bathtubs of water consumed.

6. Conclusions

Based on this analysis, I conclude that I will drink between 4 and 1,725 bathtubs of liquid over the course of my lifetime. The most likely estimate for the answer to this question is 215.5 bathtubs of liquid.

One possible source of error in my computations is that my bath tub is not a perfect rectangular solid.

One interesting fact that I learned during this investigation is that (according to MyFoodDiary.com) most doctors recommend drinking 8 to 12 glasses of water per day.

Two other formulas for calculating the amount of liquid (according to the web page) are:

0.5 ounces $\times$ Body Weight in Pounds = Daily Fluid Requirement in ounces

0.034 ounces $\times$ Daily Caloric Intake = Daily Fluid Requirement in ounces.

Another direction that I could take this investigation is to consider how much fluid I receive from foods.

Common Core State Standards

This lesson incorporates all eight of the standards for mathematical practice described in the Common Core State Standards. In addition, the following content standards may be covered (depending on the specific Fermi Questions chosen).

4.Focus.1 Develop understanding and fluency with multi-digit multiplication and division.

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, capacity, masses, and money, including problems involving simple fractions or decimals and problems that require use of simple unit conversions.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

5.Focus.2 Fluently add, subtract, multiply, and divide multi-digit numbers and make sense of standard algorithms with decimals using models for fractions and decimals.

5.Focus.3 Develop an understanding of volume.

5.OA.1 Use parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

5.MD.1 Convert among measurement units within a given measurement system.

5.MD.3 Recognize volume as an attribute of solid figures and understand the meaning of volume in terms of unit cubes.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world problems involving volumes of rectangular prisms.

6.Focus.1 Solve ratio and rate problems and understand related concepts.

6.Focus.2 Exhibit conceptual understanding and algorithmic fluency with rational numbers and the four operations.

6.Focus.3 Write, interpret, and use expressions and equations.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems (make tables, solve unit rate problems, work with percents, and convert measurement units).

6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithms.

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

6.EE.4 Identify when two expressions are equivalent and understand what equivalence of expressions implies.

6.EE.6 Use variables to represent numbers and write expressions when solving real-world or mathematical problems.

7.Focus.1 Develop understanding of and apply proportional relationships.

7.Focus.3 Solve problems involving scale drawings and informal geometric constructions, and solve problems involving surface area and volume.

7.RP.2 Recognize and represent proportional relationships between quantities.

7.RP.3 Use proportional relationships to solve multi-step ratio and percent problems.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems.

7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest.

8.Focus.1 Formulate and reason about expressions and equations.

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

HS.A.SSE.1 Interpret expressions that represent a quantity in terms of its context.

HS.Modeling Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

HS.S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.